where

$$b_{11} = -\cos\alpha_2 l_2[\alpha_1{}^2(EI)_1 \cos\alpha_1 l_1 + (1/\alpha_1)k \sin\alpha_1 l_1] + \alpha_1\alpha_2 \sin\alpha_1 l_1 \sin\alpha_2 l_2(EI)_1 - \cos\alpha_1 l_1[\alpha_2{}^2(EI)_2 \cos\alpha_2 l_2]$$

$$b_{21} = -(\sin\alpha_2 l_2/\alpha_2)[\alpha_1{}^2(EI)_1 \cos\alpha_1 l_1] - \cos\alpha_2 l_2\, [\alpha_1(EI)_1 \sin\alpha_1 l_1] - \cos\alpha_1 l_1\, [\alpha_2(EI)_2 \sin\alpha_2 l_2] - (\sin\alpha_2 l_2/\alpha_2)[k \sin\alpha_1 l_1/\alpha_1]$$

$$\begin{matrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{matrix}$$

$$b_{42} = k[(\alpha_2 l_2 - \sin\alpha_2 l_2)/\alpha_2{}^3(EI)_2] + 1$$

At station 2, the deflections $y_{2L} = y_{2R}$ are both equal to zero. Expand Eq. (8) for the y_{2L} term and then express φ_0 in terms of y_0.

The matrix equation (8) reduces to

$$\begin{vmatrix} S \\ M \\ \varphi \\ y \end{vmatrix}_{2L} = \begin{vmatrix} b_{12} - b_{11}(b_{42}/b_{41}) \\ b_{22} - b_{21}(b_{42}/b_{44}) \\ b_{32} - b_{31}(b_{42}/b_{41}) \\ 0 \end{vmatrix} \begin{vmatrix} y_0 \end{vmatrix} \qquad (9)$$

From Fig. 1, the total shear at station 2 is $R_2 + S_2$. In matrix form, this becomes

$$\begin{vmatrix} S \\ M \\ \varphi \\ y \end{vmatrix}_{2R} = \begin{vmatrix} c_{11} & 1 \\ c_{21} & 0 \\ c_{31} & 0 \\ c_{41} & 0 \end{vmatrix} \begin{vmatrix} y_0 \\ R_2 \end{vmatrix} \qquad (10)$$

In a similar manner, the transfer matrix between station $2R$ and 3 is

$$\begin{vmatrix} S \\ M \\ \varphi \\ y \end{vmatrix}_{3} = \begin{vmatrix} \cos\alpha_3 l_3 & -\alpha_3 \sin\alpha_3 l_3 & -\alpha_3{}^2(EI)_3 \cos\alpha_3 l_3 & 0 \\ \dfrac{\sin\alpha_3 l_3}{\alpha_3} & \cos\alpha_3 l_3 & -\alpha_3(EI)_3 \sin\alpha_3 l_3 & 0 \\ \dfrac{1 - \cos\alpha_3 l_3}{\alpha_3{}^2(EI)_3} & \dfrac{\sin\alpha_3 l_3}{\alpha_3(EI)_3} & \cos\alpha_3 l_3 & 0 \\ \dfrac{\alpha_3 l_3 - \sin\alpha_3 l_3}{\alpha_3{}^3(EI)_3} & \dfrac{1 - \cos\alpha_3 l_3}{\alpha_3{}^2(EI)_3} & \dfrac{-\sin\alpha_3 l_3}{\alpha_3} & 1 \end{vmatrix} \begin{vmatrix} S \\ M \\ \varphi \\ y \end{vmatrix}_{2R} \qquad (11)$$

Multiplying matrix Eq. (10) by matrix Eq. (11), the following is obtained:

$$\begin{vmatrix} S \\ M \\ \varphi \\ y \end{vmatrix}_{3} = \begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \\ d_{41} & d_{42} \end{vmatrix} \begin{vmatrix} \varphi_0 \\ R_2 \end{vmatrix} \qquad (12)$$

where

$$d_{11} = c_{11} \cos\alpha_3 l_3 - c_{21}\alpha_3 \sin\alpha_3 l_3 - c_{31}[\alpha_3{}^2(EI)_3 \cos\alpha_3 l_3]$$

$$d_{21} = (c_{11} \sin\alpha_3 l_3/\alpha_3) + c_{21} \cos\alpha_3 l_3 - c_{31}[\alpha_3(EI)_3 \sin\alpha_3 l_3]$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

$$d_{42} = (\alpha_3 l_3 - \sin\alpha_3 l_3)/\alpha_3{}^3(EI)_3$$

At station 3 for a fixed end $\varphi_3 = y_3 = 0$; thus in matrix form

$$\begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} d_{31} & d_{32} \\ d_{41} & d_{42} \end{vmatrix} \begin{vmatrix} y_0 \\ R_2 \end{vmatrix} \qquad (13)$$

For other than a trivial solution, the system in Eq. (13) has solutions different from zero if the determinant of the system vanishes. Expanding Eq. (13), the buckling load (P) may be determined when

$$d_{31}d_{42} - d_{41}d_{32} - 0 \qquad (14)$$

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[3] Marguerre, K., "Vibration and stability problems of beams treated by matrices," J. Math. Phys. **35**, 28–43 (April 1956).

Free Vibration of a Damped Semi-Elliptical Plate and a Quarter-Elliptical Plate

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Nomenclature

x, y = rectangular coordinates, in.
a, b = major and minor axes of the elliptical plates, in.
w = deflection of the plate, in.
h = thickness of the plate, in.
D = $Eh^3/12(1 - \nu^2)$ = flexural rigidity, lb-in.
E = Young's modulus of elasticity
ν = Poisson's ratio
ρ = mass density of the material, lb-sec^2-in.$^{-4}$
k = damping coefficient, lb-sec-in.$^{-1}$
t = time
w = natural frequency of the system, rad/sec

Subscripts

t, tt, n = derivatives with respect to n and t

An ordinary product solution and the Galerkin method are used as outlined by Stanisic[1] and McNitt[2] to compute the lowest natural frequency of the normal modes of free vibration of a semi-elliptical and a quarter-elliptical plate, both of which are clamped on their boundaries. The classical small-deflection theory is assumed to be valid, and the influence of rotatory inertia is neglected.

Formulation and Solution of the Problem

Because of the shape of the boundaries of the plates considered, difficulties arise for integral transform techniques. However, in the aerospace and ship industries, plates of various shapes occur. For this reason the following approximate solution is given.

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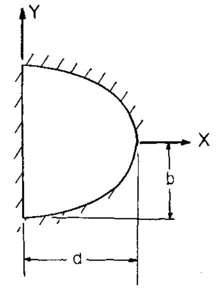

Fig. 1 Geometry of semi-elliptical plate

The motion of the plates (Figs. 1 and 2), assuming that the damping forces are proportional to the velocity, is governed by the following partial differential equation:

$$\nabla^4 w(x_1y_1t) + (k/D)w,_t(x_1y_1t) + (\rho h/D)w,_{tt}(x_1y_1t) = 0 \quad (1)$$

Choosing an ordinary product solution such as

$$w(x_1y_1t) = \phi(x_1y)e^{-\alpha t} \cos\omega t \quad (2)$$

and noting that Eq. (1) must hold for all time t, one obtains

$$\alpha = k/2\rho h \quad (3)$$

and

$$\nabla^4\phi(x_1y) - \lambda^2\phi(x_1y) = 0 \quad (4)$$

where

$$\lambda^2 = (k^2/4\rho hD)[1 + (2\rho h\omega/k)^2] \quad (5)$$

or

$$\omega = [(D\lambda^2/\rho h) - (k^2/4\rho^2h^2)]^{1/2} \quad (6)$$

Note that a typographical error in the value of ω appears in a previous paper by the author.[2] It is given correctly in Eq. (6).

Thus, if λ can be found, the natural frequency can be calculated. Let

$$\phi(x_1y) = \sum_{m=1}^{n} A_m\phi_m(x_1y) \quad (7)$$

where the ϕ_m are characteristic functions chosen so that they satisfy the geometrical boundary conditions of the problem. For a clamped plate, these conditions are that the deflection $w(\Gamma) = 0$ and the slope $w_{1n}(\Gamma) = 0$ on the boundary (Γ).

Using the Galerkin technique, it follows that

$$\iint_{\text{area}} L[\phi(x_1y)]\phi_i(x_1y)dxdy = 0 \quad (8)$$

where

$$L(\phi) = \nabla^4\phi - \lambda^2\phi \quad (9)$$

For the semi-elliptical plate, cutting the series off at two terms, the characteristic function ϕ is chosen to be

$$\phi(x_1y) = A_1\phi_1 + A_2\phi_2 = A_1x^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)^2 + A_2x^4\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)^3 \quad (10)$$

For the quarter-elliptical plate, cutting the series off at two

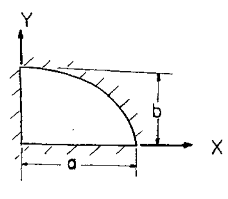

Fig. 2 Geometry of the quarter-elliptical plate

Table 1 λ for different shape ratios a/b and $a = 1$

a/b	Semi-ellipse λ	Quarter-ellipse λ
1.0	34.5	56
1.2	37.5	69
1.5	43.5	93
3.0	104.0	306
5.0	257.0	820
10.0	990.0	3200

terms, the characteristic function ϕ is chosen to be

$$\phi(x_1y) = B_1\phi_1 + B_2\phi_2 = B_1x^2y^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)^2 + B_2x^4y^4\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)^2 \quad (11)$$

By substituting Eqs. (11) and (10) in Eq. (8) and setting the determinate of the coefficients of Eq. (8) equal to zero, one obtains for the semi-elliptical plate

$$\lambda^2 = (867/a^4)[1 + 0.271(a^2/b^2) + 0.111(a^4/b^4)] \quad (12)$$

For $a = b = 1$, $\lambda = 34.5$. For the quarter-elliptical plate, one obtains

$$\lambda^2 = (1105/a^4)\{(a^2/b^2) + .917[1 + (a^4/b^4)]\} \quad (13)$$

For $a = b = 1$, $\lambda = 56$. Some values of λ for different shape ratios are given in Table 1.

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Development of a Stable "White" Coating System

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THE development of stable "white" coatings for space vehicle temperature control currently is receiving considerable attention in the aerospace industry. Recent optical reflection measurements performed on Corning #7941, multiform fused silica indicated that the bulk material had an unusually low solar absorptance of 0.08 and a high total hemispherical emittance of 0.77. The reason for this desirable combination of optical properties led to the development of a practical coating system with similar properties.

The multiform fused silica is essentially a conglomeration of fine particles of ultrapure, fused silica that has been sintered to form a free standing piece. The "whiteness" of the bulk material may be explained from the theory of optical scattering. The mixture of two optical media having different indices of refraction, with at least one having physical dimensions of the same order as the wavelength of light to be scattered, is the basic feature of a scattering layer. Corning #7941, multiform fused silica consists of a mixture of fine particles of fused silica (index of refraction = 1.46) and air (index of refraction = 1.0).

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